

The Mathematics Education I and II Courses' Effect on Teacher Candidates' Development of Number Sense

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Abstract

The purpose of this research is to examine the number sense performance of the classroom teacher candidates taking the Mathematics Education I and II courses. Moreover, it investigates whether there is a change in the number sense performance of the teacher candidates following the Mathematics Education I and II courses. Embedded experimental design was used a mixed methods research design. Pretest-posttest weak experimental design was used in the quantitative part of the study, and a case study for the qualitative part. A total of 74 teacher candidates in the third year of the Faculty of Education, Department of Classroom Teaching at a state university participated in the study. As a data collection tool, the 17-question number sense test was used in both quantitative and qualitative part of the research. The quantitative data showed that there is a significant increase in the number sense performance of teacher candidates after the Mathematics Education I and II courses. The qualitative data indicated that prior to the Mathematics Education I and II courses, the teacher candidates considered mathematics as a course in mathematical operations. They generally tended to use routine rules and algorithms in the number sense test, could not fully conceptualize some mathematical rules and phrases they use, and tried to compute instead of using number sense skills. It was found that after the Mathematics Education I and II courses, there was a decrease in the number of teacher candidates computing, whereas an increase in the number that use number sense.

Keywords: Number sense • Flexibility in calculation • Operation • Number size • Reference point • Classroom teacher candidate

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Number sense has been recognized as one of the major objectives of the elementary school mathematics in the “Everybody Counts” document created by the National Research Council in 1989 (National Research Council [NRC], 1989). In addition, the importance of number sense in mathematics education has been much more emphasized by specifying it as a standard in the document, “Curriculum and Evaluation Standards for School Mathematics,” presented by the National Council of Teachers of Mathematics (Chow, 2001; Markovits & Sowder, 1994; National Council of Teachers of Mathematics [NCTM], 1989). Along with these documents, teachers, researchers and curriculum writers have taken a growing interest in number sense (Hope, 1989; Howden, 1989, Markovits & Sowder, 1994; McIntosh, Reys, & Reys, 1992; Pike & Forrester, 1996; Reys & Yang, 1998).

In the NCTM’s “Principles and Standards for School Mathematics” (2000), number sense is identified as one of the fundamental ideas of mathematics for students. This document indicates that students need to: “(i) understand numbers, ways of representing numbers, relationships among numbers, and number systems; (ii) understand the meanings of operations and how they relate to one another, and (iii) compute fluently and make reasonable estimates” (p. 32). It points out that, in mathematics courses, children are mostly directed to use standard algorithms, and that mathematical education does not support the development of number sense (Reys et al., 1999).

Although Turkey’s Elementary Education (First-Fifth Grades) Mathematics Curriculum¹ (Ministry of National Education- Milli Eğitim Bakanlığı [MoNE], 2009) contains a “Numbers” learning topic, there are no direct statements regarding number sense. Only the descriptions in the operational estimation section under the subject of estimation strategies can be associated with number sense. Moreover, some information concerning the use of reference points is provided in the special numbers section (pp. 17-18). The Middle School (Fifth-Eighth Grades) Mathematics Curriculum was renewed in 2013. Two of its indicators for reasoning skill in mathematics are related to the results of operations, and the indicators for measurement considering a reference point are associated with number sense (MoNE, 2013, p. v).

¹ Since the implementation of the 4+4+4 educational system, a separate curriculum has not been prepared for the new first–fourth grade elementary schools. Instead, the learning acquisitions of the Elementary School (First–Fifth Grades) Mathematics Curriculum continue to be used. Therefore, this research considers the Elementary School (First–Fifth Grades) Mathematics Curriculum.

Number Sense

Although it is difficult to precisely define number sense, different researchers have offered various definitions. Hope (1989) has described number sense as a feeling of being able to make reasonable estimations about the various uses of numbers, being able to recognize arithmetic errors, being able to select the most effective computing method, and being able to notice number patterns. Number sense is also defined as “a good intuition about the numbers and their relationships” (Howden, 1989, p. 11). Reys and Yang (1998), on the other hand, defined number sense as “a person’s general understanding regarding numbers and operations” (pp. 225-226). In fact, they argue that there should be flexibility in the use of this concept, which they define as the ability and tendency to make mathematical decisions and to develop useful strategies for numbers and operations. Number sense is also defined as “using numbers in a flexible manner, thinking practically about operations with numbers, choosing the most effective and convenient solutions, sometimes finding nonstandard solutions to problems, benefiting from a reference point that simplifies the problem, and using conceptual thinking for fractions and different forms of representation for fractions” (Kayhan Altay & Umay, 2013, p. 251). Greeno (1991) claims, “number sense is a term that requires a theoretical analysis, rather than a definition” (p. 170) and that number sense is a form of cognitive expertise. According to him, number sense means knowing what resources are offered by an environment, how to find these resources in activities and how to use them, and to understand and comprehend hidden patterns. Furthermore, researchers reported that children with good number sense can do mental computation (Trafton, 1992), computing estimation (Bobis, 1991), determining the size of the numbers (Sowder, 1988), recognizing part-whole relationships and the concept of digits (Fischer, 1990), and problem solving (Cobb et al., 1991).

Although there are many definitions of number sense, the consensus in the literature is that this sense cannot be taught directly (Greeno, 1991; McIntosh, 1998). On the contrary, this sense is gradually developed by discovering numbers, visualizing them in various contexts, associating them in ways that are not restricted to traditional algorithms (Howden, 1989). Number sense starts with the relationships between the numbers from 0 to 20. However, the right intuition for numbers ends in this number interval. Children continue to enhance their number sense when they use

numbers in their operations, make sense of digit values, form flexible computing methods and use their estimation skills for given number systems (Van de Walle, Karp, & Bay-Williams, 2013).

Related Research

In most of studies done with students at different grade levels, the use of number sense in students was found to be very low (Harç, 2010; Kayhan Altay, 2010; Menon, 2004; Mohamed & Johnny, 2010; Singh, 2009; Yang, 2005). Some of these studies indicated that the structures of questions and the way questions are asked also affect students' number sense. It has been found that when the questions asked to students encourage them to think (Kayhan Altay, 2010) and are presented in a context, rather than basic operations (Sturdevant, 1991), this increases their use of number sense. It was found that students have more difficulty with questions involving rational numbers, fractions and decimals (Doğan & Yeniterzi, 2011; Işık & Kar, 2012; Kayhan Altay, 2010; Mohamed & Johnny, 2010; Singh, 2009; Uça, 2014). In addition, the research reveals a positive and significant relationship between number sense and mathematics achievements (Harç, 2010; Jordan, Glutting, & Ramineni, 2010; Kayhan Altay, 2010; Mohamed & Johnny, 2010; Sturdevant, 1991). It was reported that while responding to number sense tests, students tried to solve problems using standard operations and rules and usually chose these methods to solve questions (Harç, 2010; Kayhan Altay, 2010; Singh, 2009; Yang, 2005). It was also observed that a high rate of students who use standard methods could not remember the rules, remembered them wrong, or answered by using false generalizations such as "multiplication makes bigger" and "division makes smaller" (Harç, 2010). It has been found that students' computing performances are higher than their number sense performances (Reys & Yang, 1998; Yang & Huang, 2004), and that computing success does not necessarily accompany meaningful learning (Yang & Huang, 2004). In comparisons of number sense performances by class level, the number of students in the sixth, seventh and eighth grades who succeed in the number sense test are low. These studies reported that as the grade level rose, the number of students who succeeded in the number sense test increased (Işık & Kar, 2011), and students' number sense rises with age from ages 6 to 11 (Pike & Forrester, 1996).

Although there are many studies that examine the mathematical performance of elementary school students, only a few studies focus on their teachers. The limited number of studies investigating the mathematical sense of primary

school teachers and teacher candidates show that they have many weaknesses: using mathematical rules incorrectly, not understanding the real meaning of mathematical concepts, and generally not being ready to teach mathematics (Cuff, 1993). The educators responsible for training these teachers can design their courses more effectively if they know which skills they possess and which skills they lack. If the perceptive and conceptual errors of teacher candidates are known in advance, their teaching performance can be enhanced by eliminating those errors during their training.

Johnson (1998) found that there are gaps in teacher candidates' understanding of rational numbers, and when they encounter non-standard problems, they focus on the use of algorithms. These misunderstandings of rational numbers are due to a lack of understanding of the various representations of rational numbers. Rasch (1992) and Hungerford (1994) attribute these difficulties with rational numbers to the fact that they do not fully comprehend this number system and its characteristics.

Teachers are required to understand primary school mathematics in depth in order to teach mathematics effectively (Ball, 1990). Studies have shown that teacher candidates know the operations in the elementary school mathematics; however, they do not fully understand them conceptually (Ball, 1990; Kılcan, 2006; Ma, 1999; Zazkis & Campbell, 1996). Some research indicates that teacher candidates have poor number sense (Kayhan Altay & Umay, 2011; Tsao, 2005; Yaman, 2012; Yang, Reys, & Reys, 2009). They are especially prone to use standard algorithms (Ball, 1990; Newton, 2008; Şengül, 2013; Thanheiser, 2010; Yang, 2007). It has been found that although teacher candidates' tendencies to standard computational techniques continue after training, number sense can be improved with proper training (Kaminski, 2002; Markovits & Sowder, 1994). Moreover, the research reports that a significant improvement in the number sense of teacher candidates was observed when they attend the courses that focus on the development of number sense (Nickerson & Whitacre, 2010; Whitacre, 2007; Whitacre & Nickerson, 2006).

The Importance and Aim of the Study

The importance of number sense is mentioned indirectly in all mathematics curricula, but learning acquisitions for its improvement are not included explicitly. It is thought that number sense skills of teacher candidates could positively or negatively

affect the number sense perception, ability and performance of students. Since number sense is indicated as a skill that should be learned by first-fourth grade students, the number sense of first-fourth grade teacher candidates is important. This study was planned based on the belief that determining the number sense levels of teacher candidates needs to play an important role in teacher training programs.

In light of this information, the question emerges, “To what extent do our teacher candidates have the skills we want our students to learn?” Thus determining teacher candidates’ number sense performance and answering the question, “How do the Mathematics Education I and II courses affect the number sense performance of teacher candidates?” seems to be important.

The aim of this study is to investigate whether there is a change in the number sense performance of teacher candidates after the Mathematics Education I and II courses taught in a university according to the CoHE course definitions. In addition, it aimed to qualitatively examine the number sense performance of classroom teacher candidates before and after these courses. In the framework of these objectives, responses to the following questions are sought:

1. Does the number sense performance of classroom teaching third grade teacher candidates show a statistically significant difference before and after the Mathematics Education I and Mathematics Education II courses?
2. Does the number sense performance of classroom teaching third grade teacher candidates show a qualitative change after Mathematics Education I and Mathematics Education II courses?

Method

Model

This study used a mixed model including both quantitative and qualitative techniques. First, the qualitative data was collected. Then, the quantitative data was collected and analyzed. Later, qualitative data was collected again for the purpose of obtaining in depth information from teacher candidates about this process and to support other findings. Mixed method research done in this way is called embedded experimental design. The stages of embedded experimental design are shown in Figure 1 (Creswell & Plano Clark, 2007, pp. 68-69).

Qualitative data were collected from 10 selected teacher candidates by doing interviews before the Mathematics Education I and II Courses. The statistical analyses were performed by collecting pretest and posttest quantitative data about the participants’ number sense. Qualitative data was collected again by interviewing the 10 teacher candidates selected. These data were used to show how the number sense performances of teacher candidates changed after the Mathematics Education I and II courses.

Experimental design was used since the purpose of the quantitative part of the study was to determine the effect of the Mathematics Education I and II courses on the number sense performance of teacher candidates. This is a weak experimental design study based on a single group pretest-posttest model (Fraenkel & Wallen, 2005, pp. 251-272). Since there is only a single independent variable (the Mathematics Education I and II courses), a research design with a control group was not chosen. Due to the nature of the dependent variable (number sense performance), in which a difference would be expected to occur naturally between the groups that take or do not take the

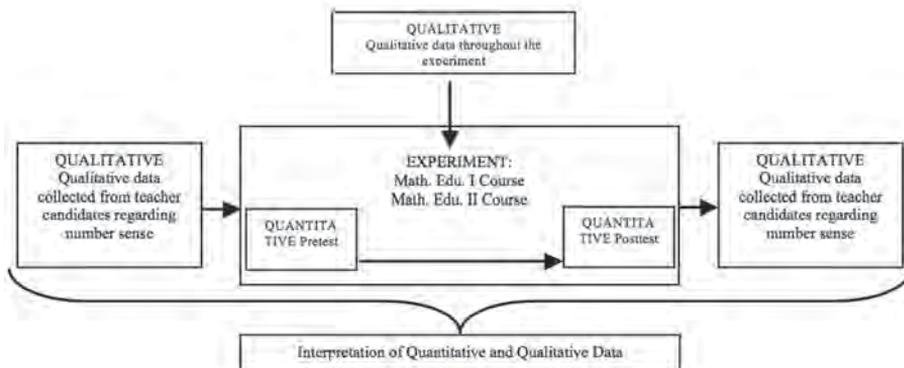


Figure 1: Embedded experimental design stages.

Mathematics Education I and II courses, a single-group research design was chosen.

Case study design was used for the study's qualitative data. In this part, the teacher candidates' ways of responding to the questions on the number sense test before and after taking the Mathematics Education I and II courses were determined by structured interviews.

Study Group

Teacher candidates in their third year of study in the Faculty of Education, Department of Elementary School Classroom Teaching at a state university were selected for the quantitative part of the study. This year of study was chosen because the mathematics education courses are taught in their third year. A total of 74 teacher candidates, 24 males and 50 females, enrolled in Mathematics Education I and Mathematics Education II during the 2011-2012 academic year, participated. Teacher candidates were not included in the study if they did the pretest but not to the posttest, or did the posttest but not to the pretest.

In the qualitative part of the study, population and sample determination was not performed, because no generalization could be made, so 10 teacher candidates who participated in the quantitative part of the study were randomly selected. An interview was conducted with these teacher candidates (6 females, 4 males) at the beginning of the fall semester before the Mathematics Education I course and at the end of the spring semester after the Mathematics Education II course.

Procedure

The content of the training given during the study was limited to the course definitions of the Mathematics Education I and Mathematics Education II courses described in the Council of Higher Education (Yükseköğretim Kurulu [CoHE], 2007) Faculty of Education Teacher Training Undergraduate Programs (see Table 1). Both of the courses were chosen as the experiment, because the subjects in the course definitions are common to both semesters. The researcher never deviated from the course content and presented the subjects in the normal sequence. To avoid causing the teacher candidates to have specific expectations, no emphasis on the concept of number sense was intentionally made in class. Information about the topics and concepts were presented in accordance with the course content order, and no

information about the study on number sense was given to the teacher candidates.

In the Mathematics Education I course, general mathematics education subject such as teaching and learning theories, mathematical knowledge, the use of models, mathematical skills, effective mathematics education and the use of technology were presented. Towards the end of the semester, topics concerning numbers, operations and education with numbers were taught. The Mathematics Education II course involved studies of fractions, operations with fractions, decimal fractions, ratios, proportions, percentages, geometry, measurement and graphs, and the teaching of these concepts.

Table 1

CoHE Faculty of Education Teacher Training Undergraduate Programs Mathematics Education I and II Course Definitions

Mathematics Education I (3-0-3)	Mathematics Education II (3-0-3)
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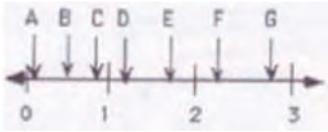
The purpose of mathematics education and basic principles; the history of mathematics education (in the world and Turkey), teaching and learning strategies to be used in mathematics education; the scope, objectives and characteristics of the elementary school mathematics program; major theories of learning and their relationships with mathematics learning; fundamental skills, association, representations, communication, reasoning, problem solving (strategies, steps, problem types, etc.) in mathematics education; using information technology, development of the number concept in children (developments before counting, one to one correspondence, cardinal value, conjugation, separation, comparison, equation, scarcity, abundance, etc.), digit value, composition and structural properties of natural numbers, arithmetic operations, relevant topics and achievements in the first, second and third grade elementary mathematics curricula and examples of relevant activities (pp. 35-36).

Fractions, student difficulties with learning fractions, different meanings of fractions, fraction models, equality, comparison, ordering, operations with fractions, decimal fractions, operations with decimal fractions, sample activities appropriate to the program's learning acquisitions, Geometry, development of geometric thinking in children, 2 and 3-dimensional geometry topics and their teaching, sample activities according to the geometry learning acquisitions of the program, measurement and measures, development of measurement in children, dimension, area, volume, time measurement, weighing, money, sample activities proper to the measuring learning acquisitions of the program, data management, charts and graphs, sample activities relevant to the data learning acquisitions of the program, measurement and evaluation in mathematics education, multiple measurement and evaluation methods and techniques (p. 37).

Data Collection

The number sense test (NST) was developed by Kayhan Altay (2010). It consists of 17 questions used to determine the number sense performance of teacher candidates (see Table 2). Validity and reliability studies were done for the test once again. Three field experts examined the test, and

Table 2
Subscales of Number Sense Test and Sample Questions

Subscales	Sample Question	Item Numbers
Flexibility in Calculation	6) Which numbers can be written into the parentheses to provide the following equation? Explain your thinking. $50 + () \div () = 65$	1, 3, 4, 6, 7, 8, 10, 13
Conceptual Thinking in Fractions	14) Which letter on the number line corresponds to a fraction the numerator of which is slightly greater than the denominator? Explain how you found it. 	11, 12, 14, 15
Use of Reference Points	Explanation: 5) Which number should be in the place of A on the following number line? 	2, 5, 9, 16, 17

reported that the test was appropriate to use with teacher candidates. After the pilot study, the Kuder-Richardson 20 internal consistency coefficient of the test was determined to be .75, so the test was accepted as reliable. The NST consists of three subscales including “Flexibility in calculation (8 items),” “Conceptual thinking in fractions (4 items)” and “Use of reference points (5 items).” Kayhan Altay has found the reliability coefficient for test measurements to be 0.86. In this study, on the other hand, the KR-20 values for pretest and posttest measurements were examined, and it was found that their values were .85 and .83, respectively. These results indicate that the pretest and posttest measurements are reliable.

The data collection tool (NST) was used in the same form for both the pretest and posttest, and the same verbal instructions were given. The pretest was administered at the beginning of fall semester. The posttest was administered at the end of spring semester. Thus, the effect of remembering the pretest while doing the posttest was reduced to a minimum. On the NST, 1 point was given to teacher candidates who solved the problem using number sense, regardless of the correctness of their answers. Zero points were given to those who solved the problem by calculating or using standard formulas, or gave incorrect responses.² The highest possible score on the test is 17, and the lowest possible score is 0. The questions answered correctly by the teacher candidates who participated in the qualitative part of the study using computation were calculated separately.

² Two of the teacher candidates who participated in the study used number sense while responding to a question, one on the seventh question and the other one on the 11th question; however, they responded incorrectly due to an operational error. One point was given for these responses.

In the qualitative part of the study, the NST used in the quantitative portion of the study was administered to the teacher candidates before and after the Mathematics Education I and II courses to see if they used number sense to answer the questions and how they did so. The interviews with each teacher candidate lasted approximately one-hour.

Data Analysis

The answers were checked and scored after the NST was administered for the quantitative data of the study. The pretest and posttest scores obtained were transferred to a computer, and statistical analyses were conducted to determine whether there was a difference between the scores. Non-parametric methods were used because the pretest and posttest data did not have a normal distribution. The effect size was also considered while doing benchmark testing.

The qualitative data from the interviews with teacher candidates were analyzed and graphed using NVivo 10 qualitative data analysis software. Content analysis was used to analyze the data. There are two different approaches to categorizing in content analysis, the closed approach and the open approach. Preexisting categories are used in the closed approach, and categories created by content analysis are used in the open approach (Bilgin, 2000, pp. 10-11). The closed approach was used to find out whether the teacher candidates used number sense and how they thought about the number systems and mathematical structure of the questions. In other words, the number systems and mathematical structures of the questions on the NST were used as themes for encoding.

To test the reliability of the study, the coding obtained from the interviews were examined with another expert working on number sense, and the items about which they agreed or disagreed were identified. Miles and Huberman (1994) formula (comparison percentage = [agreement / (agreement + disagreement)] x 100) was used for testing, and the value was found to be 92.35 for the initial interviews and 94.12 for the final interviews. Thus, the study was found to be reliable.

Results

Findings for the First Sub-Problem

Descriptive statistics for the pretest and posttest NST scores of the teacher candidates were computed for this sub-problem (see Table 3).

Table 3
Descriptive Statistics of Teacher Candidates' Pretest and posttest scores

Measurement	n	\bar{X}	sd
Pretest	74	7.04	2.68
Posttest	74	9.08	3.21

Since the pretest and posttest scores did not have a normal distribution, the non-parametric Wilcoxon signed-rank test was used to determine if there is a significant difference between these scores. Its results are shown in Table 4.

Table 4
Wilcoxon Signed-Rank Test Results of the Pretest and Posttest Scores

Posttest-Pretest	n	Mean Rank	Sum of Ranks	z
Negative Rank	5	25.80	129	-5.977*
Positive Rank	57	32.00	1824	
Equal	12			

* Based on negative rankings

The Wilcoxon signed-rank test results show that there is a statistically significant difference between the scores of the teacher candidates before and after Mathematics Education I and II courses ($z = -5.977$, $p = .00 < .05$). This difference supports positive ranking (i.e., posttest score) when considering the mean rank and sum of ranks of the difference between scores. According to Cohen r result, it can be also indicated that this difference is effective in

medium size ($r = .463$). These results show that Mathematics Education I and II courses help to develop the number sense of teacher candidates.

Findings for the Second Sub-Problem

This sub-problem concerns the teacher candidates' use of number sense on the NST. The pretest and posttest NST scores of the 10 teacher candidates who were interviewed before and after Mathematics Education I and II courses are shown in Table 5.

As Table 5 shows, all these teacher candidates used more number sense after the Mathematics Education I and II courses. Thus, the Mathematics Education I and II courses may contribute to the development of teacher candidates' number sense.

In the next section, the NST was discussed in two sections to find out how a change has occurred in the teacher candidates' responses in these two sections. Nine NST questions are operational questions and 8 concern number size. Some of these questions were given as integers, some as fractions and some as decimal fractions (see Table 6). Therefore, in this section, the NST is examined in two sections, operations and number size. In each of these sections, the results are presented under 3 sub-headings, integers, fractions and decimal fractions.

Table 6
NST Questions about Operations and Number Size

	Integers	Fractions	Decimal Fractions
Operation	4, 6, 8, 16	9, 13	1, 3, 7
Number Size	-	2, 11, 12, 14, 15, 17	5, 10

Operation Questions: These NST questions are shown in Table 6. There are 9 questions in this category, including 4 with integers, 2 with fractions, and 3 with decimal fractions. Information about the number sense scores of teacher candidates received from pretest and posttest regarding operation questions in the NST is shown in Table 7.

Table 7 shows that after the Mathematics Education I and II courses the teacher candidates used number sense more. Here is an example of an operation they did with integers:

Table 5
The Pretest and Posttest NST Scores of the Teacher Candidate Interviewees

	TC1	TC2	TC3	TC4	TC5	TC6	TC7	TC8	TC9	TC10
Pretest	8	6	5	7	13	4	12	6	5	6
Posttest	13	10	8	8	16	8	16	10	8	10

Table 7

The Pretest and Posttest NST Scores of Teacher Candidate Interviewees: Operation Questions

	TC1	TC2	TC3	TC4	TC5	TC6	TC7	TC8	TC9	TC10
Pretest	5	4	4	5	8	2	6	3	4	4
Posttest	9	5	6	4	9	4	9	6	6	7

Question 8: How do you solve the following equation easily? Explain how you did so.

“5,000,032+2,000,725+1,000,068–1,000,725”

Here are some quotes from interviews with the teacher candidates who either answered this question incorrectly before the Mathematics Education I and II courses or gave the correct answer by computing, but used number sense after the Mathematics Education I and II courses.

Before the Mathematics Education I and II Courses:

(TC7 underlined the digits in the millions.) “I would operate on the underlined numbers and add and subtract them. There would be no issue if we did the same for other remaining numbers because all of them have 7 digits, and their fourth, fifth and sixth digits are already same.” (TC 7 did normal addition and subtraction operations and wrote down 7,000,100.) (TC7)

“First, I do an operation with the millions. Then, I add the hundreds and decimals and add the result to the others.” (TC 8 wrote $(5,000,000+2,000,000+1,000,000-1,000,000)+(32+725+68+725)$. In one case, 725 should be negative. TC8 missed this and got the wrong answer.) (TC8)

After the Mathematics Education I and II Courses:

“Adding 725 and -725 eliminates them. Likewise, 1,000,000 and 1,000,000 eliminate each other. Only 5,000,000, 2,000,000 and 100 remain. Adding them gives the result, 7,000,100.” (TC7)

“(TC8 crossed out the numbers 725.) We do this without looking at the remainders, then add and subtract the hundreds, but students can make mistakes with the place of the digits.” (On one side, TC8 wrote 5,000,000, 2,000,000, 1,000,000 and 1,000,000, and on the other, TC8 wrote $32+68=100$ on a side. TC8’s final response was 7,000,100). (TC8)

Before the Mathematics Education I and II courses, most of the students attempted to solve this problem by doing computation. Here is an example of an operation they did with fractions:

“*Question 9: Which sum is greater than 1? Explain how you find the answer.*”

Before the Mathematics Education I and II courses, 4 teacher candidates responded to this question using number sense. After the Mathematics Education I and II courses, 5 teacher candidates have solved it using number sense. There was no increase in the use of number sense. Before the Mathematics Education I and II courses, 5 teacher candidates tried to find the result by finding the common denominator and doing addition. While 4 of them found the correct answer, 1 gave an incorrect answer. One teacher candidate converted the fractions into decimal fractions, and found the solution by calculating it. Here are quotes from the interviews with these teacher candidates:

Before the Mathematics Education I and II Courses:

(TC1 found the common denominator for the fractions in the c and d options and did addition. TC1 did not look at the a and b options, once the operation’s result in the d option was found to be greater than 1). “It is 49/45. It is greater than 1, because the numerator is greater than the denominator.” (TC1)

(TC10 found the common denominator and added all the options). After we find the common denominator, option d is greater than 1. (TC10)

After the Mathematics Education I and II Courses:

“5/9 is greater than 0.5. Likewise, 8/15 is greater than one-half. If I add both of them, I obtain a number that is greater than 1.” (TC1)

“I evaluate whether the fractions added are greater than 1/2 or not. The two fractions in the d option are greater than 1/2.” (TC10)

These quotes show that teacher candidates do calculations using routine algorithms without thinking about the problem. One teacher candidate found the decimal fractions that corresponding to the fractions by doing division, and added these numbers to see if their sums were greater than 1. After the Mathematics Education I and II courses, two of teacher candidates who used routine algorithms for this question used number sense in their responses. They compared the fractions with 1/2 after the Mathematics Education I and II courses. Another teacher candidate (TC4) used

number sense before the Mathematics Education I and II courses; however, this participant tried to use a routine operation after the Mathematics Education I and II courses.

There was an increase in the use of number sense after the Mathematics Education I and II courses for the 3 questions which contain an operation in decimal fractions.

Question 1: What is the shortcut solution for 0.25x16? Show how you find it.

The same 7 teacher candidates responded to this question with the number sense both before and after the Mathematics Education I and II courses. On the other hand, the 3 teacher candidates who responded incorrectly to this question before the Mathematics Education I and II courses responded correctly to this afterwards by doing computation. Here are some quotes from interviews with the teacher candidates who used number sense:

“Four units of 0.25 equals 1. Sixteen of them makes 4.” (TC1)

“Since 4 units of 0.25 represents 1, I would take 1 of them by dividing 16 to 4.” (TC5)

Of the teacher candidates who used number sense, only one of them (TC1) has responded by using the ratio and proportion in the question. The other 6 teacher candidates who used number sense stated that they solved the problem by dividing 16 by 4. Here are quotes from the interviews with the teacher candidates who answered this question incorrectly before the Mathematics Education I and II courses, but did computing after the Mathematics Education I and II:

(TC2 wrote 25/100 instead of 0.25.) “Like 25/100 multiply 16. We multiply 16 units of 25/100. Or we do repeated adding.” (TC2)

“I would convert 0.25 into a fraction and multiply it by 16.” (TC5)

“I would reach the solution by writing (25/100)x16 and simplifying 25/100 by making it 1/4.” (TC6)

These teacher candidates converted the decimal fraction into a fraction first. Then, they have reached the result by multiplying an integer with a fraction. This could be an indication that they have

a tendency to continue to use routine algorithms learned while doing operations in the decimal fractions.

Question 3: Is the result of 6,464x0.54 greater or less than 3,232? Why?

$a. \frac{3}{11} + \frac{8}{7}$ $b. \frac{7}{18} + \frac{5}{13}$ $c. \frac{1}{2} + \frac{4}{9}$ $d. \frac{5}{9} + \frac{8}{18}$

While 5 teacher candidates have responded to this question using number sense before the Mathematics Education I and II courses, 7 teacher candidates answered it using number sense afterwards. Here are quotes from interviews with teacher candidates who used number sense:

Before the Mathematics Education I and II Courses:

“If we consider 3232 as a, 2a x 0.54 means multiplying a by a number like 1,xyz (When 2 is multiplied by 0.54 a number greater than 1 is obtained). Therefore, it is greater.” (TC3)

“Multiply 54/100 by 6,464 (TC10 tried to do the operation and could not find the answer).” (TC10)

After the Mathematics Education I and II Courses:

“It is greater because 0.54 is greater than 1/2.” (TC3)

“I calculated in my mind that 0.54 is greater than 1/2. That is, if 0.50 => 1/2, then 0.54 it is greater than 1/2. So multiplying by it yields a result greater than one-half.” (TC10)

These quotes show that before the Mathematics Education I and II courses teacher candidates were prone to use routine algorithms. While TC10 tried to do multiplication operation by converting 0.54 into a fraction, TC3 tried to explain the operation in a more algebraic way. Both teacher candidates reported after the Mathematics Education I and II courses that by comparing 0.54 with 1/2, the result of the operation would be greater than 3,232.

TC 6 and TC8 tried to do the operation by multiplying 6,464 with 54/100, both before and after the Mathematics Education I and II courses. This may indicate that it is very difficult for the teacher candidates to get rid of the habit of using routine algorithms.

Number Size Questions: The numbers of the NST questions about number size and were shown in Table

Table 8
Pretest and Posttest NST Scores of the Teacher Candidate Interviewees: Number Size Questions

	TC1	TC2	TC3	TC4	TC5	TC6	TC7	TC8	TC9	TC10
Pretest	3	2	1	2	5	2	6	2	1	2
Posttest	4	5	2	4	8	4	8	5	3	3

6. There are a total of 8 questions in this category: 6 in fractions and 2 in decimal fractions. The teacher candidates' pretest and posttest NST scores on the number size questions are shown in Table 8.

Here is a number size question:

Question 2: Write a fraction between $\frac{1}{2}$ and $\frac{6}{7}$.
Explain how you found it.

Only 1 teacher candidate used number sense before the Mathematics Education I and II courses, whereas 4 teacher candidates used number sense after them. The use of computing for this question was limited to 6 teacher candidates both before and after the courses. The interviews reveal that 2 out of 3 increases after the courses came from teacher candidates who gave incorrect answers, and only 1 teacher candidate switched to number sense from computation. The other 5 teacher candidates who did computation also tried to answer the question by computing after the Mathematics Education I and II courses. This outcome shows that teacher candidates could not abandon the rule of finding a common denominator and comparing the numerators. Here are quotes from the interviews with teacher candidates after the Mathematics Education I and II courses:

"I want to obtain a common denominator, $7/14$ and $12/14$. We combined the numbers from 7 to 12 with the same size. In this case we can write $8/14$." (TC3)

"I would find a fraction between $7/14$ and $12/14$ by equalizing the denominators." (TC6)

The interviews with other teacher candidates who used computing show that they all tried to do comparisons by finding a common denominator. No one compared by equalizing the numerators. This outcome suggests that although there is more than one rule for comparing fractions, there is always a tendency to use the most common method. Here is a quote from the response of a teacher candidate who used number sense after the Mathematics Education I and II courses:

"I write $3/4$. The first fraction, $1/2$, represents the half of a whole, and $6/7$ represents a number that is close to a whole. I chose $3/4$, because I had to choose a number between a half and a whole." (TC5)

While 3 out of 4 teacher candidates who used number sense after the Mathematics Education I and II courses responded to this question in similar ways, 1 teacher candidate made a comparison by using unit fractions.

"I choose 5 units of $1/7$ as the answer because it is between $1/2$ and 6 units of $1/7$. That is how I find the number between them." (TC4)

Question 14: Which letter on the number line corresponds to a fraction the numerator of which is slightly greater than the denominator? Explain how you found it.



Explanation:

Three teacher candidates answered this question using number sense before the Mathematics Education I and II courses, whereas 7 answered this question using number sense afterwards. Before the courses, only 3 teacher candidates indicated that if the numerator of a fraction is greater than its denominator, that fraction is greater than 1. Furthermore, since it was stipulated that the numerator needs to be slightly greater, D was selected since it is the letter closest to 1. This question is about the display of fractions on the number line. A number line is an abstract model. Before the Mathematics Education I and II courses, the number sense of the teacher candidates was weak because they lacked information about the display of fractions on a number line.

One teacher candidate had understood the concept of compound fractions before the Mathematics Education I and II courses, where the very similar wording, combined fraction, was used. Although students did not know the exact name of the concept, they could answer such questions using number sense if the concept was understood.

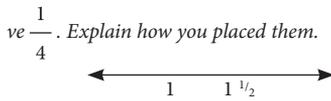
In other words, although teacher candidates had issues using number sense with fractions on a number line before the Mathematics Education I and II courses, this issue has been overcome after the Mathematics Education I and II courses. Most of the teacher candidates used their number sense by stating that in this question the fraction needs to be compound and the numbers on numerator and denominator should be close to 1.

"It is the d option since it is closer to the one." (TC4)

"It is the d option. It is close to 1 and greater than 1." (TC6)

"It is d. If a compound fraction is slightly greater, it should be greater than and closer to 1." (TC8)

Question 15: Considering the points given at the above number line, place the fractions of $\frac{1}{2}$, $2\frac{1}{2}$



Two teacher candidates responded to this question using number sense before the Mathematics Education I and II courses. One of them placed the points on the number line model accurately, and then tried to match the given fractions with these points. The other teacher candidate, on the other hand, placed these fractions on the number line by considering into which interval the given numbers fall.

Although TC1 has used number sense on question 14, TC1 did not use it on this question and responded incorrectly.

“Two whole $1/2$, 2 times 1 whole $1/2$.” (TC1)

This teacher candidate did not have any problem placing the compound fractions, could not use number sense with the mixed fraction. This outcome may be due an inability to establish a relationship between the compound fraction and the mixed fraction. Another teacher candidate was unable to fully understand the mixed fractions.

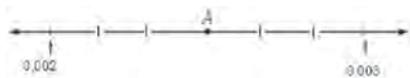
“Two whole $1/2$, 2 times $1/2$.” (TC2)

Another teacher candidate converted the fractions in the question into decimal fractions, thinking decimal fractions better, and tried to place these decimal fractions, but gave an incorrect answer.

In this question, use of number sense after the Mathematics Education I and II courses has increased only 1, and this number has remained at 3. Although this was a question asking to place fraction on the number line as in the previous question, teacher candidates who could use number sense in the other question could not use their number senses in this question.

Here is question 5 about number size in decimal fractions:

Question 5: Which number should be placed at point A on the following number line? Why?



Before the Mathematics Education I and II courses, 3 teacher candidates responded correctly to this question using number sense. The other 7 teacher candidates either answered this question incorrectly or left it blank. None of the teacher candidates found the correct result by calculating.

(TC3 wrote $0.003-0.002=0.001$) “We take half of it. That is 0.0025.” (TC3)

“The point A is just the middle point. Therefore, it must be 0.002 plus 0.0003 divided by 2.” (TC7)

“Since 0.002 equals 0.0020; so if other numbers between these two numbers are 0.0021, 0.0022 and so on up to 0.0029, then the middle is 0.0025.” (TC8)

This question, like questions 14 and 15, uses a number line model, but this question is about the display of decimal fractions on a number line. The teacher candidates generally gave incorrect answers to this question before the Mathematics Education I and II courses. The interviews with these teacher candidates show that they usually erred when they tried to convert the decimal fractions into fractions.

After the courses, the number of teacher candidates who used number sense for question 5 increased from 3 to 6. These teacher candidates generally looked at 0.002 as 0.0020, and 0.003 as 0.0030, and then responded accordingly.

“The answer is 0.0025. I took its arithmetic mean.” (TC2 took the arithmetic mean of 0.0020 and 0.0030.) (TC2)

“Since 0.002 is 0.0020 and 0.003 is 0.0030, the number between them is 0.0025.” (TC7)

“It is 0.0025, because there are 0.0020 and 0.0030.” (TC10)

Discussion and Conclusion

Studies of number sense usually research the current status of teacher candidates (Kayhan Altay & Umay, 2011; Şengül, 2013; Yaman, 2012; Yang, 2002). This study tried to see if and how teacher candidates' number sense performance is changed by taking the Mathematics Education I and II courses. Since this research has both quantitative and qualitative findings, the conclusion is divided into two sections.

Discussion of Quantitative Findings and Conclusions

This study found a significant difference in favor of the posttest in the number sense performance of third year teacher candidates after taking Mathematics Education I and II courses. This result shows that their number sense performance was enhanced by the Mathematics Education I and II courses.

Various researchers have examined the effect of number sense training with different age groups: preschool (Diezmann & English, 2001), elementary school (Zaslavsky, 2001; Yang, 2003), middle school (Markovits & Sowder, 1994; Yang, 2002), high school and undergraduates (Kaminski, 2002; Tsao, 2005; Whitacre, 2007). In this research, like in this study, it was found that training does develop number sense. In almost all of these studies, training was provided to support the development of number sense.

These studies' common feature is the positive effect of training in number sense on the number sense of students. This study also found that the number sense of teacher candidates improved after the Mathematics Education I and II courses. Most of the training given in these studies is related to number sense. In this study, although the training is not intended to focus number sense, in the course definitions of the Mathematics Education I and II courses, skills such as number sense, estimation, and mental computation are mentioned since they are required by the Elementary School (First-Fifth Grades) Mathematics Course Curriculum (2009). In addition, number sense topics such as number systems and operations with them are also included in these courses. This situation can be seen as an evidence that, although the training is not specifically about number sense, courses in which information about number sense, numbers and operations is given can lead to the development of number sense.

The highest possible score on the NST is 17. The students' mean score is 7 before the Mathematics Education I and II courses and approximately 9 afterwards. Although this difference is statistically significant, 9 is still a low score for prospective mathematics educators. The concept of number sense is included in the Mathematics Education I and II courses in a limited fashion. The low 2 point difference between the pretest and posttest scores may be due to these limitations.

Discussion of Qualitative Findings and Conclusions

Data from the interviews with teacher candidates were presented as questions about operations and questions about number size on the NST. Therefore, these two parts are also examined separately in the discussion.

Operations: The interviews with teacher candidates showed that teacher candidates used number sense in the questions containing addition and subtraction operations with integers before and after the Mathematics Education I and II courses. For questions that required mental computation

and included addition, subtraction, multiplication, it was found that before the Mathematics Education I and II courses, teacher candidates tried to do computation and were prone to use routine rules and formulas. After the Mathematics Education I and II courses, on the other hand, there was an increase in the use of number sense on questions required mental computation, and the tendency to do computation with formulas and rules persisted. In research conducted by Yang (2007), it was found, like this study, that two-thirds of the teacher candidates who participated in the study and had not received training on number sense tried to obtain results by doing computation instead of using number sense. Similarly, in the test prepared by Tsao (2005), it was also reported that in interviews with teacher candidates who had high or low scores, especially low-achieving students used standard written methods and rule-driven solutions. Another study conducted by Reys et al. (1999) with students between the ages of 8 and 14 in four different countries revealed that students generally choose written computation. The researchers attributed this to the fact that mathematics curricula stress computation. In light of these results, it appears that the results obtained by this study are consistent with the research done with different age levels.

Students in mathematics programs that stress computation always perceive the mathematics as a course about operations (Greer, 1997). Conceptual information about mathematical concepts is overtaken by operational information (Yaman, Toluk, & Olkun, 2003). Therefore, students or teacher candidates who do not receive any number sense may always feel the need to do computation for mathematical problems. The new Middle School (9th-12th Grades) Mathematics Course Curriculum (MoNE, 2013, p. 1) says:

“Instead of operational and information-oriented mathematics education, mathematical concepts are introduced to classrooms in discussions, and operational and conceptual information are handled in a balanced manner.”

If this is done, future teacher candidates will come to universities with conceptual knowledge and not merely operational information.

Four teacher candidates responded to the question about addition and subtraction with fractions using number sense before the Mathematics Education I and II courses. This number increased to 5 after the courses. The teacher candidates who participated in the interviews also used the rule, “when two

fractions with unequal denominators are added or subtracted, the denominators are equalized and numerators are added or subtracted," which is known and used by almost all of the students without questioning. For the question about the multiplication and division of fractions, only 2 of the teacher candidates used number sense before the Mathematics Education I and II courses, whereas this number rose to 7 after the courses. This question asks about multiplying and dividing the same number with a simple fraction. Before the Mathematics Education I and II courses, some of the teacher candidates used the rule, "multiplication makes numbers larger, and division makes them smaller," while others used, "to divide fractions, the dividing fraction is multiplied after being reversed." After the Mathematics Education I and II courses, the numbers of teacher candidates who used these algorithms decreased significantly. Yang's study (2007) indicated that teacher candidates usually found a common denominator when comparing two fractions, and added that when teacher candidates are asked to use a different method, they reported that they did not know any other way. Tsao (2005) indicated that the most difficult section of the NST for teacher candidates was the problems on fractions. In their study, Reys et al. (1999) directed the students to use a reference point when working with fractions, and eventually found that students had misconceptions about fractions and estimation. Yaman (2012) found that teacher candidates had the most problems using number sense with fractions. Similarly, this study has also revealed that teacher candidates had the most difficulties in the sections of the NST with fractions.

The interviews with the teacher candidates show that before the Mathematics Education I and II courses the use of number sense for questions about operations with decimal fractions was high. There was a slight increase in these numbers after the Mathematics Education I and II courses. Before the Mathematics Education I and II courses, they use a multiplication algorithm converting decimal fractions into fractions. In addition, one teacher candidate understood a multiplication operation using a proportion. It was observed that before the Mathematics Education I and II courses, 4 teacher candidates responded incorrectly to the question about adding decimal fractions or used computation. However, all the teacher candidates used number sense for this question after the Mathematics Education I and II courses. Thus, it can be said that before the Mathematics Education I and II courses, the teacher candidates lacked

operational knowledge about the addition of decimal fractions. Like these results, Singh (2009) found that Malaysian students had difficulties assigning meanings to rational numbers and decimal fractions. In fact, it was found that students tried to answer using addition rules for estimation questions with decimal fractions. It was also specified that students tended to use algorithms and rules. Suh, Johnston, Jamieson, and Mills (2008) did an experimental study by giving lessons with mathematical demonstrations. The researchers found that after these lessons, fifth and sixth grade students improved their use of number sense with decimal fractions. Specifically, the demonstrations with the hundreds table led students to report that they understood decimal fractions better and recognized that decimals fractions are an extension of the decimal system.

Number Size: On the NST, 4 of the 6 questions about number size are comparisons of fractions. Before the Mathematics Education I and II courses, the teacher candidates used their number sense least for these questions. For these questions the teacher candidates generally tried to compare the fractions by finding common denominators. None of the teacher candidates who participated in the interviews used fraction comparison algorithms to equalize the numerators. The reason for this may be that, although the students learn multiple algorithms, they have a tendency to use the most frequently used algorithms. After the Mathematics Education I and II courses, there was a decrease in the use of algorithms and an increase in the use of number sense. Like this study, Yang (2002) indicated that, after cooperative learning training, some of the sixth grade students have used denominator equalizing method, and some of them have used shape demonstration method for questions about comparing fractions. Yang (2002) encouraged students to draw shapes and visually compare the fractions. Moreover, the researcher claimed that class discussion after cooperative learning training and activities helped students to overcome difficulties with fractions and argued that number sense can be enhanced by communication and discussion.

A change was found after the Mathematics Education I and II courses in the responses to a question about fractions on a number line. This question asked the teacher candidates to find which a point on the number line was only slightly larger than 1. The number intervals and the location of the points on the number line are obvious. It

was found that while teacher candidates could use their number sense for this question after the Mathematics Education I and II courses, no change was detected in their use of number sense for another question about the placement of fractions on the number line after the Mathematics Education I and II courses. This question asked teacher candidates to place the other fractions given on two fraction number lines. The number line model is an abstract model of fractions (Olkun & Toluk Uçar, 2012) and was not fully comprehended by the teacher candidates. The teacher candidates were able to use their number sense to work on a number line with clear points and intervals. However, for the question in which only the interval between 1 and $1/2$ was given and they were asked to find the other points, they remained at the same level in terms of number sense after the Mathematics Education I and II courses. This outcome shows that the teacher candidates have problems with creating number line models on their own, and they need all its elements to be able to use it. Studies have indicated that students have difficulty displaying fractions on number lines (Doğan Temur, 2011; Ersoy & Ardahan, 2003; Pesen, 2008). Similar results were found in another study of teacher candidates (Toluk Uçar, 2009). Research by Bay (2001) focused on the importance of using number line activities in courses and designed training in number line activities to improve number sense. This training was administered to middle school students by the researcher, and afterwards it was concluded that the number line activities are a highly effective tool for comparing the size of numbers and fractions, which is also consistent with the results of this research.

The number size questions with fractions included on the NST concern the display of two items on models. One of these questions asks to display the given fraction on the model, and the other question asks which fraction can be associated with the given model. It was found that the use of number sense increased for both questions after the Mathematics Education I and II courses. The teacher candidates who tried to use routine algorithms before the Mathematics Education I and II courses abandoned this tendency after the Mathematics Education I and II courses. In their study, Yang and Huang (2004) examined the differences between the results of computation test, a pictorial display test, a symbolic display test and a number sense test administered to sixth grade students. For example, an operation with fractions was asked in the computation test; and again, in the pictorial display test, they were asked to draw the same operation as pictorial demonstration

format or as a modeled version. They found that the computation test was the one in which students were most successful. Compared to the computation test, the students had little success on the other tests. Based on these findings, the researchers have emphasized that computation skills can be effective, especially when it includes mathematical understanding. Indeed, the teacher candidates responded to the questions with computation, without having training, have added conceptual knowledge along with their operational knowledge through training, and due to this conceptual knowledge, they often used number sense.

Singh (2009) has argued that student achievements on the questions about multi-display are higher than questions about number concepts. Similarly, it was observed in this study that teacher candidates used number sense more for display questions about fractions, than for operation and comparison questions.

The other two questions about number size use decimal fractions. The interviews show that the use of number sense for them increased considerably after the Mathematics Education I and II courses. One of the questions is about the placement of decimal fractions on the number line. In this question, it was found that before the Mathematics Education I and II courses, teacher candidates usually tried to answer the question by converting decimal fraction to fractions, and most of them gave incorrect answers. On the other hand, after the Mathematics Education I and II courses, it was found that the teacher candidates could do these conversions more easily and used number sense more. In their study, Markovitz and Sowder (1994) taught seventh grade students to compare fractions by helping them to discover the relationships between fractions and decimal fractions. It was determined that the students who participated in this training were more successful in fraction comparisons. These results are also consistent with the results of this study.

When the teacher candidates were asked to compare 9 decimal fractions, they were unable to do so before the Mathematics Education I and II courses. This outcome can be seen as an indication that before the Mathematics Education I and II courses, the teacher candidates had conceptual inadequacies in decimal fraction comparisons. However, after the Mathematics Education I and II courses, more teacher candidates responded to this question correctly by using number sense.

Recommendations

The results of the study show that teacher candidates are especially prone to use rules and algorithms and perceive mathematics as a class about operations. Therefore, lessons for them should be planned where they can perceive the conceptual aspects of mathematics. These teacher candidates will begin teaching and pass their own perceptions to the students they teach. Consequently, teaching them the conceptual aspects of mathematics is necessary, if we want to prevent students from only perceiving the operational aspects of mathematics.

Activities for teacher candidates should be performed for them to understand why number sense is important. In addition, information about number sense and its components should be given, particularly in the Mathematics Education I and II courses.

The results of the study show that the Mathematics Education I and II courses increased the number sense performance of teacher candidates. Nevertheless, the mean score obtained by the teacher candidates is 9, and since the highest score to be achieved is 17, 9 is a low score. Therefore, more

number sense activities should be included in their courses by modifying the content of Mathematics Education I and II. Moreover, an elective course can be added to the curriculum, in which information about number sense, its components, how it can be developed, computation, estimation and mental computation skills can be offered. Finally, the changes in the number sense performance of teacher candidates before and after this elective course can be examined.

Especially in the Basic Mathematics I and II courses, questions that include mental computation and enhance estimation skills should be given to teacher candidates, instead of questions that require constant computation with paper and pencil.

The results of the study confirm that teacher candidates' use of number sense with fractions is problematic, particularly, with integer fractions. Furthermore, it was found that teacher candidates had difficulties with the number line model, which is an abstract model for displaying fractions. Therefore, special attention should be given to these matters in the Mathematics Education I and II courses.

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