Does Mastery of Goal Components Mediate the Relationship between Metacognition and Mathematical Modelling Competency?

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Abstract

Prior studies suggested close correlations among metacognition, mastery goal, and mathematical modelling competency. The present study examines the relationship between metacognition and mastery goal that may influence mathematical modelling competency. The current study employs 538 students of a mathematics education program; among these students, 483 (89.8%) are males and 55 (10.2%) are females, aged from 18 to 22 years old. The study follows a correlational research design to investigate and measure the degree of relationship among mathematical modelling competencies, mastery goal, and metacognition. Findings indicate that mastery goal positively affects mathematical modelling competency. SEM analysis indicates significant and positive influence of task- and self-approach goals on mathematical modelling competency, whereas task-avoidance goals are significantly and negatively related to mathematical modelling competency. By contrast, self-avoidance goals did not affect mathematical modelling competency. Task-approach goal is a positive partial mediator, task-avoidance goal is a negative partial mediator, self-approach goal is a positive full mediator, and self-avoidance goal is not a mediator between metacognition and mathematical modelling competency. In conclusion, metacognition positively affects the mathematical modelling competency of students, which is influenced by task-approach, task-avoidance, and self-approach goal but not self-avoidance goal.

Keywords

Confirmatory factor analysis • Mastery goal • Mathematical modelling competency • Metacognition • Structural equation modelling
The presence of mathematical modelling competency is an important component in mathematics that embraces arithmetic, algebra, geometry, and calculus (Dundar, Gokkurt, & Soylu, 2012). Mathematical modelling competency allows students to have ability of identifying, mathematizing, interpreting and validating, as well as the capability of making analysis or comparison (Blomhoej & Jensen, 2003; Blum, Galbraith, Henn, & Niss, 2007; Maaß, 2006). The educational researchers highlighted a large number of benefits of modelling competencies as key factors in the study of intricacy and modern science (English, 2008; Gainsburg, 2006; Kartal, Dunya, Diefes-Dux, & Zawojewski, 2016). Students in higher education level who hold these competencies are expected to be successful in conducting research because these competencies comprise rigorous scientific procedure (Haines & Crouch, 2010). Interestingly, mathematical modelling is considerably employed by government and manufacturer for guiding and even making their decisions (Hunt, 2007). Hence, application and modelling in a mathematics classroom received strong support from educational researchers in the last few decades (Niss, Blum, & Galbraith, 2007).

The modelling process is initially believed to be difficult (Czocher, 2017; de Oliveira & Barbosa, 2013; Hidayat & Iksan, 2015; Jupri & Drijvers, 2016; Mentzer, Huffman, & Thayer, 2014; Wijaya, Heuvel-panhuizen, Doorman, & Robitzsch, 2014; Yew & Akmar, 2016). In a study conducted by Blomhøj and Kjeldsen in 2013, students encountered problems about mathematising the expression ‘proportional to the square of population size’ before finding the formula $N' = kN^2$. Despite the huge challenge of teaching mathematical modelling, limited research was conducted on why mathematical modelling competency is difficult to learn and how certain factors might influence competency. The survey research by Yildirim (2010), Frejd and Ärlebäck (2011), Mischo and Maaß (2012), Sharma (2013) and the comparative study in German by Schukajlow, Krug, and Rakoczy (2015) are exceptions.

Prior studies suggested other potential factors influence students, such as goal orientation (Topcu & Leana-Tascilar, 2016) and metacognition (Galbraith, 2017; Gabriele Kaiser & Stender, 2013; Stillman, 2011). These two factors are part of the definition of mathematical modelling competency (Biccard & Wessels, 2011). However, only a few studies documented the relationship among these variables in mathematical modelling competency. To our knowledge, the effects of metacognition and mastery goal on mathematical modelling competency of students have not yet been tested. The current study focuses on the direct and indirect effects of the relationship between metacognition and mathematical modelling competency. The indirect effects are the mediating effects of four mastery goal components of task- and self-approach and task- and self-avoidance. We extend existing mathematical modelling competency literature by discussing these complex relationships in the real-life problems for students of mathematics education programmes. The research questions guiding the current research are the following:
1. Do metacognition and mastery goal directly affect mathematical modelling competency?

2. Are the four mastery goal components a mediator between metacognition and mathematical modelling competency?

Background Literature

Mathematical Modelling Competency

Modelling, which is also known as mathematising or mathematisation (Niss, 2015), refers to the process of organising representational descriptions (Lesh & Lehrer, 2003), where symbolic meaning and the formal structures of the model emerge (Greer & Verschaffel, 2007). Maass (2006) identified modelling as a competency, which is richly linked to the modelling process and highly emphasised in research on modelling (Mehraein & Gatabi, 2014; Yilmaz & Tekin-Dede, 2016). However, to date, the meaning of mathematical modelling competency is not obvious in mathematics because of different views. According to Biccard and Wessels (2011), mathematical modelling competency is defined through three different aspects of cognitive, affective, and metacognitive competencies. Affective and metacognitive competencies are no longer considered positive side impacts, but significant constituents of mathematical modelling competency. In the present study, the definition of mathematical modelling competency refers to the cognitive dimension. To simplify assumptions, clarify the objective, formulate the issue, and assign variables, establish parameters and constants, formulate mathematical expressions, choose a model, interpret graphic, link to the real context (Haines & Crouch, 2001) are known as mathematical modelling competency, which are also referred to as micro assessment (Houston, 2007).

The two main perspectives of teaching mathematical modelling are modelling as a method and as content (Galbraith, 2007, 2012; Julie, 2002). The rationale for modelling as a method concentrates on the ways in which modelling has goals of introducing other curricular material and connected priorities or to enable learners to study (Galbraith, 2012). In mathematical modelling as content, Julie (2002) insisted that specific mathematical knowledge should be applied in a real world context. Learning and teaching modelling skills require criteria which are both internal and external to education (Galbraith, 2012). In other words, the end goal of this view is that students should have modelling competency in which they apply mathematical concepts and procedures in natural and social phenomena. Therefore, mathematical modelling competency is referred to in content perspectives.

A standard framework for mathematical modelling has not yet been agreed upon. Modelling has been used variously in literature (e.g., Blomhoej & Jensen, 2003; Blum...
& Leiß, 2005; Ferri, 2006; Galbraith, Stillman, & Brown, 2010; Galbraith & Stillman, 2006; Kaiser & Schwarz, 2006; Lange, 2006; Lesh & Doerr, 2003; Shahbari & Peled, 2017; Sokolowski, 2015; Verschaffel, Greer, & De Corte, 2002). These processes differ from each other because of distinctive perspectives (Blomhøj, 2009; Kaiser & Sriraman, 2006), but they usually offer a visual display of phases. These modelling processes are classified into six perspectives, namely, realistic modelling; contextual modelling; educational modelling; socio-critical modelling; epistemological or theoretical modelling, and meta-perspectives (Haines & Crouch, 2010). This study falls under the educational perspective on mathematical modelling.

**Mastery Goal**

Mastery goal is an achievement goal or ability. Mastery goals (adaptive) are reflected through challenge pursuit and efficient perseverance in the deal with barriers (Stout & Dasgupta, 2013). Focusing on mastery goals requires comparison between previous and current achievement, which then develops as self-reference focused on results in performance situations (Poortvliet, Janssen, Van Yperen, & Van de Vliert, 2007). Mastery goal is a useful side of the learning process (Bonnett, Yuill, & Carr, 2016) that predicts achievement (Dompnier et al., 2015), affects performance (Phan, 2014), and leads to success in problem-solving strategies (Dweck & Leggett, 1988). Mathematical instruction based on mastery-oriented usually encourages better teamwork and readiness to work collaboratively (Bonnett et al., 2016). Mastery-oriented students are self-regulated; they use self-monitoring and organizational approaches and are adaptive to failure assignment (task goal). By contrast, mastery avoidance goal, which is the most recent addition to the model, aims to avoid misconceptions, and not mastering tasks. The approach uses standards of not being erroneous and not making the task incorrectly. The latest model of achievement goal, namely, the 3 x 2 achievement goal model, divides into mastery goal that concentrates on the achievement of task-based competence ands on specific problems (McCollum & Kajs, 2007). Point goal orientation in mathematical teaching refers the process of focusing on mathematical needs of students in terms of particular competencies, which can be developed using diverse mathematical topics, rather than concentrating on the subject matter (Khait, 2003).

The mastery goal framework is distinguished into approach and avoidance focus (Elliot & McGregor, 2001; Elliot, 1999). The mastery approach goal focuses on mastering assignment, learning, and understanding. This approach employs standards of self-advancement, progress, and exhaustive concept of self-based competence and mastery avoidance, which concentrates on avoidance of task-based incompetence or self-based incompetence (Elliot, Murayama, & Pekrun, 2011). A task-approach goal refers to the achievement of task-based competence, whereas task-avoidance goal pays attention on the avoidance of task-based incompetence. The focus of self-based
competence is one’s intrapersonal track as the evaluative referent. Students who are involved in self-approach goal intend to improve their performance, whereas students involved in self-avoidance goals aim not to demonstrate performance show worse than they previously performed (Wynne, 2014).

The presence of mastery goal is useful in mathematical modelling because tasks are usually believed as a group activity (Houston, 2007). Mastery goals influence student relations, such as teacher–student relations, peer inclusion and conflict (Polychroni, Hatzichristou, & Sideridis, 2012), and interest in the activity (Senko & Harackiewicz, 2005). Mastery-oriented students assess collaboration with respect to contribution to learning, friendship, and class cohesion; they tend to be ready to collaborate with counterparts regardless of their social group affiliation (Levy, Kaplan, & Patrick, 2004). In addition, Hagstrom and White (2006) reported that success in solving problems is richly linked to shared talk. This finding reflects the weightiness of socially shared conversation in the development of problem-solving approaches. According to Ferri and Lesh (2013), the modelling cycle can be managed to become more goal-oriented if students learned to speak and imagine about mathematical concepts and their means of comprehending mathematics.

**Relationship between Mastery Goal and Mathematical Modelling Competency**

The positive relationship between mastery goals and mathematical modelling competency is obtained from considerable research in other domains. Research demonstrates that mastery goal correlates positively with the academic achievement (Chen, 2015; Dompnier et al., 2015; Dompnier, Darnon, & Butera, 2013; Gul & Shehzad, 2012; Mirzaei, Phang, Sulaiman, Kashefi, & Ismail, 2012; Sideridis, 2005; Yeung, Craven, & Kaur, 2012) and problem-solving success of students (Gardner, 2006). In the 3 x 2 achievement goal framework, task-based goals predict academic self-concept (Méndez-Giménez, Cecchini-Estrada, Fernández-Rio, Saborit, & Méndez-Alonso, 2017), examination performance (Stoeber, Haskew, & Scott, 2015), perceived competence (Mascret, Elliot, & Cury, 2015), and material absorption (Elliot et al., 2011). Self-based goals are not connected to material absorption (Elliot et al., 2011) and perceived competence (Mascret et al., 2015), whereas self-based and self-avoidance goals require more help-seeking (Yang, Taylor, & Cao, 2016). However, self-based goals are predictors of academic self-concept (Méndez-Giménez et al., 2017). To our knowledge, the relationship between mastery goals and metacognition on students’ mathematical modelling competency has not yet been explored. We hypothesised that task- and self-approaches positively affect mathematical modelling competency, whereas task- and self-avoidance goals negatively affect mathematical modelling competency.
Metacognition

Metacognition involves psychological and cognitive concepts (Papaleontiou-Louca, 2008) and is defined as the knowledge or activity of people about their cognitive processes and products or anything associated with these concepts (Flavell, 1976). According to Flavell’s (1979) model, metacognition is indicated by four major aspects, namely, metacognitive knowledge, experiences, goals, and actions (or approaches). Metacognitive knowledge contains knowledge or belief factors, namely, person, task, and strategy, which serve and intercommunicate to affect the course and result of cognitive enterprises. In relation to modelling competency, Stillman (2011) provided examples of related factors in metacognitive knowledge. As a modeller, person factor can be illustrated with consciousness of difficulty in easily formulating plausible estimates. Example of task factor pertains to consciousness of task characteristics that affects task solution, whereas the strategy factor refers to consciousness of their effectiveness when used in the past. However, metacognitive knowledge about teaching processes might be right or wrong, and this self-knowledge is usually invulnerable to transformation (Veenman, Van Hout-Wolters, & Afflerbach, 2006).

Given that metacognition involves the process of managing and coordinating, solving problems include complex activities, such as various cognitive operations (Garofalo, Lester Jr., 1985). Metacognition activity guides students to select approaches to assist comprehend the problem, plan courses of action, monitor execution action while using approaches, evaluate the results of approaches, and revise or abandon non-productive approaches (Brown, 1978). For example, a modelling cycle can be used to identify the kind of treatment required to tackle certain barrier (Stillman, 2011). According to Lingefjärd (2011), metacognitive competencies that overarch the process of mathematical modelling have to be involved in a model for barrier s and chances.

Relationship between Metacognition and Mathematical Modelling Competency

Stillman, Galbraith, Brown, and Edwards (2007) provided a metacognitive model of modelling competency in mathematical modelling competency. Metacognitive modelling competencies refer to the capability and agreement to observe and reflect about students’ own modelling cycle based on metacognitive knowledge (Kaiser & Stender, 2013). This metacognitive activity can be viewed forwards and backwards regarding steps in the modelling cycle (Galbraith, 2013). Research findings suggested that the use of metacognitive strategy is useful in mathematical modelling and problem solving. Expert’s success and students’ failure result from the presence and absence of productive “metacognitive” behaviour (Schoenfeld, 1983), such as poor metacognition that can prevent problem solving (Schoenfeld, 2007).

Metacognition is the most important strategy related to mathematical achievement (Bonnett et al., 2016; Callan, Marchant, Finch, & German, 2016; Özcan, 2016; Tzohar-
Does Mastery of Goal Components Mediate the Relationship between Metacognition...

Rozen & Kramarski, 2014) and problem solving skills (Yusnaeni & Corebima, 2017). Studies confirmed the importance of metacognition in improving mathematical modelling competency. Metacognition influences the development of modelling strategy of learners when the effects of four metacognitive components are taken into consideration (i.e., awareness, planning, cognitive strategy, and self-checking) (Yildirim, 2010). Students who demonstrated improved self-checking abilities indicate increased modelling competency growth. Cognitive strategy and planning skills mediate modelling competency development. After several experiences with modelling, students with increased skills in these two metacognitive dimensions improved their modelling skills. However, the cognitive and metacognitive activities did not sequentially happen in the process. Instead, they were simultaneously established and twisted in the modelling cycle (Hidiroğlu & Bukova Güzel, 2016). Hence, we hypothesised that metacognition positively influences mathematical modelling competency.

**Relationship between Mastery Goal and Metacognition**

Researchers indicate that mastery goals are closely associated with students’ metacognition (Gardner, Jabbour, Williams, & Huerta, 2016; Gul & Shehzad, 2012; King & Mcinerney, 2016; Mirzaei et al., 2012). Student characteristics related to mastery goal orientation can be self-directed by using self-monitoring and organizational approaches; they are also adaptive to failures on specific problems (McCollum & Kajs, 2007). Bonnett et al. (2016) hypothesised that utilizing a mastery approach goal within a mathematics curriculum promotes metacognition, increases motivation, and assists students reach an underlying knowledge of mathematical concepts, thereby enhancing mathematics achievement. In addition, students provided with learning goals have better metacognition and higher engagement (Gardner et al., 2016).

Zafarmand (2014) found that among three components of goal orientation, mastery goal has a positive effect on metacognitive awareness planning and monitoring. Students utilizing good mastery approach goal hold better metacognition rather than those with performance goals. In the same fashion, students’ views of the mastery and performance goals had significant relationship with metacognitive self-regulation (Kadioglu & Kondakci, 2014). A few studies illustrated mastery goals as a mediator in academic achievement (Chen, 2015; Diseth & Kobbeltvedt, 2010). Surprisingly, limited study corroborated that task- and self-approach goals as well as task- and self-avoidance are mediators. Hence, we hypothesised that mastery goal, which involves task- approach goals and self-approach goals and task-avoidance goals and self-avoidance goals, mediates between metacognition and mathematical modelling competency.
Method

Participants and Procedure

This study follows a relational survey model to investigate and measure the degree of relationship among mastery goal, metacognition, and mathematical modelling competency (Codd, 1970). The relationships among mastery goal, metacognition, and mathematical modelling competency were measured using structural equation modelling analysis (SEM) (Byrne, 2012). A priori model which integrates variables in this study is developed by theories and previous studies (Figure 1). There are three main variables, namely mastery goal, metacognition, and mathematical modelling competency in which the correlation between these variables is indicated by straight arrows. Based on literature, the model combining these variables has not been tested previously and the fit of this model is assessed using structural equation modeling (SEM). Students with mastery goals will perform well metacognition, which eventually influence their mathematical modelling competency. In addition, metacognition is also hypothesized to have indirect impact modelling competency.

The population in this study comprised students of a mathematics education program in Indonesia. Populations were selected because of the mathematics course taken and the modelling experiences commonly found in mathematics education program. For example, participants should have registered for advanced courses, such as calculus, geometry, linear algebra, linear program, and statistics. Thus, the assumption is they have implicitly learnt the process of mathematical modelling competency. Cluster random sampling was appropriate because the current research selected groups rather than individuals (Fraenkel & Wallen, 2009). The current study employed 538 mathematics education program students in Indonesia. The number of female participants was 483 (89.8%), whereas male participants were 55 (10.2%) with ages ranging from 18 to 22 years old. The gender disproportion in the departments of mathematics education program resulted in a substantial proportion of female students. The academic years of the targeted students were the first until
the fourth year from 2017–2018. However, the current research only used the first, second, and third years because the fourth academic year students were in practical session. The students enrolled in the first academic year were 133 (24.7%), the second academic year participants were 223 (41.4%), and the third academic year participants were 182 (33.8%). They completed the questionnaire that covers 22 items in a mathematical modelling test, 20 items in metacognitive inventory, and 12 items in the $3 \times 2$ achievement goal questionnaire.

### Measures

**Mathematical Modelling Test.** The mathematical modelling test was originally developed by Haines and Crouch (2001) and includes the following items: “simplify assumptions regarding the real world task,” “clarify the goal of the real model,” “formulate a proper task,” “assign variables, parameters, and constants in a model on the basis of sound understanding of model and situation,” “formulate pertinent mathematical expressions representing the problem addressed,” “choose a model,” and “interpret and connect the mathematical solution to the real world context.” Each correct answer for multiple-choice items was awarded 2 points, and partial credit were awarded 1 point. Wrong answers were awarded 0 points. A total of 22 questions were used in the mathematical modelling test, which had a maximum score of 44. In addition, an item response analysis was utilized to indicate the discrimination and the difficulty indices (Ariffin, 2008; Hambleton, Swaminathan, & Rogers, 1991), while the most commonly utilized measurement models used for adaptive tests fall within the framework of Item Response Theory (IRT). IRT in general defines a probabilistic relationship associating item and test taker traits to the possibility of endorsing every single of the response categories for that item. Since there are different IRT model, the three-parameter logistic model (3PL) was suitable because it had been created to include difficulty ($b$), discrimination ($a$) and randomness ($c$) or guessing parameters (Hambleton, Swaminathan, & Rogers, 1991). Item’s difficulty is the index of students answering correctly (Ariffin, 2008). Ariffin (2008) also defines the discrimination index as value to show whether an item can distinguish between low and high performance students. Items are acceptable when they can distinguish two groups of students. The discrimination and the difficulty indices for all questions including correct answer, partial credit and wrong answer were calculated by the Winsteps software. By using Rasch model, the item difficulty value ranges from $+0.50$ to $-1.00$ logits. It exceeds the acceptable value of $+3.00$ to $-3.00$ logits and is assumed good (Linacre, 1994), in which 19 items are medium level while three items are easy level. The discrimination indices of each item of the mathematical modeling test ranged from 24.55% to 57.27%, which indicates that 2, 13, and 7 items had fairly good, good, and very good discrimination indices, respectively. Moreover, by using the binomial probability theorem, it is easy to deduce that the probability to guess 10 right answers is around 0.0045 (Lingefjärd
& Holmquist, 2005). Therefore, each item to test students’ mathematical modeling competency were retained in the actual study. Moreover, measurement model of mathematical modeling competency was also provided.

The reliability value of the mathematical modeling test was good (0.82) (Tavakol & Dennick, 2011). Moreover, in the current research, two are two types of validity which are: content and construct validity. To ensure content validity, researcher did not remove any item for the each instrument. The instrument also was reviewed by several experts from several universities. It was assessed by expert team of two mathematics expert in which one expert is from Universitas Syiah Kuala (Unsyiah) and another is from University of Malaya (UM). For metacognition and achievement goal instruments, the items were reviewed by expert team of two psychology education where one expert is from Universitas Gadjah Mada (UGM) and another is from University of Malaya (UM). Content validity also involve the wording and format of the items on a scale which is consistent with the construct of interest. In addition, Confirmatory Factor Analysis (CFA) was performed to determine construct validity of the instrument that also means identifying any underlying association between the items on the scale. All composite reliability (CR) values of mathematical modelling competency components ranged from 0.69 to 0.78 and exceeded the 0.6 desirable standards. This finding indicated high internal consistency. The average variance extracted (AVE) of the eight latent variables ranged from 0.50 to 0.63 and exceeded the 0.5 common cut-off value, which demonstrated that the current research presents acceptable discriminant validity. Therefore, each mathematical modelling competency item in this study was retained for use in testing the students.

3 x 2 Achievement Goal Questionnaire. The instrument was adopted from Elliot et al. (2011) and involves six components classified into mastery goals (i.e., task approach, task avoidance, self-approach, and self-avoidance) and performance goals (i.e., other-approach and other-avoidance goals). However, the current study only measured mastery goal using task approach, task avoidance, self-approach, and self-avoidance. The questionnaire consists of six questions that reflect the two components. A seven-point Likert-type scale ranging from 1 (“strongly disagree”) to 7 (“strongly agree”) was employed to measure the 3 x 2 achievement goal questionnaire (Gillet, Lafrenière, Huyghebaert, & Fouquereau, 2015). Reliability values of certain scales exceeded the 0.70 desirable standard (task-approach goal, $\alpha = 0.88$), (task-avoidance goal, $\alpha = 0.74$), (self-approach goal, $\alpha = 0.93$), and (self-avoidance goal, $\alpha = 0.93$). All CR values of the mastery goal components ranged from 0.75 to 0.93 and exceeded the 0.6 desirable standards. This finding indicated high internal consistency. The AVE of the four latent variables ranged from 0.50 to 0.83 and exceeded the 0.5 common cut-off value, which demonstrated that this study presents acceptable discriminant validity.
Metacognitive Inventory Questionnaire. O’Neil and Abedi (1996) originally
devolved the metacognitive inventory, which Yildirim (2010) modified and used in
mathematical modelling competency. The instrument involves four sub-constructs
comprising 20 statements, with five statements per sub-construct. The sub-constructs
of the instrument are awareness (e.g., “I am aware of what modelling strategies to use and
when to use them to solve an exercise”), cognitive strategy (e.g., “I attempt to discover
the main ideas in an exercise”), planning (e.g. “I try to understand the goals of an exercise
before I attempt to solve it”), and self-checking (e.g. “I check my accuracy as I proceeding
through the solution”). A five-point Likert-type scale with responses of strongly disagree
(1), disagree (2), uncertain (3), agree (4), and strongly agree (5) was used to measure the
metacognitive inventory questionnaire. Cronbach’s alpha internal consistency reliabilities
of the four metacognition sub-constructs were above the $\alpha > 0.70$ minimum common cut-
off (awareness, $\alpha = 0.83$; cognitive strategy $\alpha = 0.85$; planning $\alpha = 0.84$; self-checking, $\alpha
= 0.83$). All CR values of the metacognition sub-construct ranged from 0.83 to 0.85 and
exceeded the 0.6 desirable standard, which indicated high internal consistency. The AVE
of the four latent variables ranged from 0.50 to 0.54 and exceeded the common cut-off
value of 0.5, which demonstrated that this study presents acceptable discriminant validity.

Data Analysis

This study considered a large number of data screening-related issues, such as
handling missing data, multi-collinearity, and identification of outliers and normality
using the Statistical Package for the Social Sciences (SPSS) 23.0 software before
conducting further analysis. Outliers were identified through a boxplot for each
sub-construct. The benchmark of the univariate normality of the construct in a
measurement model for a latent variable is that the skewness and kurtosis values
of each item ranged from $-1.96$ to $+1.96$ at the 0.05 significant level (Hair, Black,
Babin, & Anderson, 2010). Finally, the correlation matrix with correlations more
than 0.90 is regarded as multi-collinearity (Kline, 2005).

CFA procedures using AMOS 18.0 were employed to explore whether the
established dimensionality and the factor-loading pattern fit the Indonesian context.
According to Awang (2012), goodness-of-fit is evaluated through chi-square ($\chi^2$)
($P > 0.05$), comparative fit index (CFI > 0.90), Tucker Lewis index (TLI> 0.90),
and root mean-square error of approximation (RMSEA < 0.08). Cronbach’s alpha
coefficients, CR, AVE, and split-half correlations were computed to determine the
reliability of the instrument (total and sub-constructs). Alpha values in the current
research were not expected to be comparatively high. According to Hair et al. (2010),
alpha values of 0.60 to 0.70 in exploratory research are satisfactory. CR should be
higher than 0.60 and AVE should be more than 0.50 (Awang, 2012). To determine
the extent to which a mediator affected the total effect of the outcome variable, the
significance of indirect effects was examined using the Sobel test.
Results

Preliminary Analysis

The amount of missing data in the current research varied from 0 to 0.5% per item and the missing data are random (MCAR) (Kline, 2005). The means, standard deviations, correlation matrix, and the skewness and kurtosis for all variables are listed in Table 1.

Table 1

<table>
<thead>
<tr>
<th>Variable</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<tbody>
<tr>
<td>1. Metacognition</td>
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<td></td>
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<td></td>
<td></td>
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<tr>
<td>2. Task-approach goal</td>
<td>0.264**</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Task-avoidance goal</td>
<td>0.303**</td>
<td>0.406**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Self-approach goal</td>
<td>0.343**</td>
<td>0.524**</td>
<td>0.506**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5. Self-avoidance goal</td>
<td>0.301**</td>
<td>0.379**</td>
<td>0.529**</td>
<td>0.520**</td>
<td></td>
</tr>
<tr>
<td>Skew</td>
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<td>-0.17</td>
<td>-0.92</td>
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</tr>
<tr>
<td>Kurtosis</td>
<td>1.83</td>
<td>0.58</td>
<td>1.26</td>
<td>1.63</td>
<td>1.90</td>
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<tr>
<td>Mean</td>
<td>3.90</td>
<td>4.87</td>
<td>5.42</td>
<td>5.60</td>
<td>5.29</td>
</tr>
<tr>
<td>SD</td>
<td>0.39</td>
<td>0.96</td>
<td>1.03</td>
<td>0.98</td>
<td>1.06</td>
</tr>
</tbody>
</table>

** Correlation is significant at the 0.01 level (2-tailed).

Table 1 reveals that a medium level of correlation (0.264 to 0.529) exists between the constructs. Metacognition was positively correlated with task-approach goal \((r = 0.264)\), task-avoidance goal \((r = 0.303)\), self-approach goal \((r = 0.343)\), and self-avoidance goal \((r = 0.301)\). Task-approach goal was positively correlated with task-avoidance goal \((r = 0.406)\), self-approach goal \((r = 0.524)\), and self-avoidance goal \((r = 0.379)\). Task-avoidance goal was positively correlated with self-approach goal \((r = 0.506)\) and self-avoidance goal \((r = 0.529)\). Finally, self-approach goal was positively correlated with self-avoidance goal \((r = 0.520)\). This correlation indicates that the discriminant validities of the variables were reached because the correlation matrix yielded correlations less than 0.90 (Kline, 2005). In terms of univariate normality, skewness values for metacognition, task-approach, self-approach, task-avoidance, and self-avoidance goal ranged from -1.06 to -0.17, whereas kurtosis values for metacognition, task-approach, self-approach, task-avoidance, and self-avoidance goal ranged from 0.58 to 1.90, which indicated normal distribution. The mean values varied among variables, with metacognition at M = 3.90 and SD = 0.39, task-approach goal at M = 4.87 and SD = 0.96, task-avoidance goal at M = 5.42 and SD = 1.03, self-approach goal at M = 5.60 and SD = 0.98, and self-avoidance goal at M = 5.29 and SD = 1.06.
Testing the Measurement Models

CFA procedures were used to confirm the factorial validity of variables. The metacognition measurement model resulted in acceptable model fit at $\chi^2 = 191.35$, $\chi^2/df = 1.60$, CFI = 0.980, TLI = 0.990, and RMSEA = 0.033. The mastery goal measurement model indicated acceptable model fit at $\chi^2 = 72.926$, $\chi^2/df = 1.519$, CFI = 0.994, TLI = 0.992, and RMSEA = 0.031. The mathematical modeling competency measurement model indicated acceptable model fit at $\chi^2 = 248.485$, $\chi^2/df = 1.373$, CFI = 0.976, TLI = 0.969, and RMSEA = 0.026.

Testing the Hypothetical Structural Model

Outcomes of the SEM analysis in the present study revealed the hypothetical structural model at $\chi^2 = 1880.491$, $\chi^2/df = 1.552$, RMSEA = 0.032, TLI = 0.924, and CFI = 0.928. All evaluations resulted in acceptable model fit for the Indonesian context. All factor loadings of the four metacognitions and the four mastery goal components ranged from 0.63 to 0.74 and from 0.65 to 0.93, respectively. The factor loading values exceeded the 0.50 desirable standard (Hair et al., 2010). Table 2 shows that the hypothetical structural model is excellent.

Table 2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^2$</td>
<td>1880.491</td>
</tr>
<tr>
<td>$\chi^2/df$</td>
<td>1.552</td>
</tr>
<tr>
<td>RMSEA</td>
<td>0.032</td>
</tr>
<tr>
<td>TLI</td>
<td>0.924</td>
</tr>
<tr>
<td>CFI</td>
<td>0.928</td>
</tr>
</tbody>
</table>

Note. $\chi^2$: Chi-square goodness of fit; df: Degrees of freedom; CFI: Comparative Fit Index; TLI: Tucker-Lewis Fit Index (TLI); RMSEA: Root Mean Square Error.

In addition, the model of CFA presented in Figure 2 became the finalized model that indicated relationships among metacognition, mastery goal, and mathematical modelling competency in the Indonesian context. The final model derived from the current research can be used as an alternative in explaining the prior study on the relationships between metacognition, mastery goal, and mathematical modelling competency.
Note. Insignificant regression paths are removed from the model.

Figure 2. Final model of the study.

Relationships between Metacognition and Mathematical Modelling Competency

We assumed that metacognition goal positively affects mathematical modelling competency. Significant relationships exist between the two constructs ($\beta = 0.441$, $t = 5.106$, $p < 0.05$). Thus, students who utilise metacognition performed well in mathematical modelling competency were fully confirmed. Metacognition is one of the factors contributing to mathematical modelling competency.
Relationships between Mastery goal and Mathematical Modelling Competency

We hypothesised that task- and self-approaches positively affected mathematical modelling competency, whereas task- and self-avoidance goals negatively affected mathematical modelling competency. The task-approach ($\beta = 0.045, t = 3.230, p < 0.05$), self-approach ($\beta = 0.032, t = 2.035, p < 0.05$), and task-avoidance goals ($\beta = -0.069, t = -3.997, p < 0.05$) affected the mathematical modelling competency of students. However, self-avoidance goals ($\beta = -0.022, t = -1.616, p = 0.106$) did not affect their mathematical modelling competency. Thus, H2 is not fully supported. Task- and self-approach goals of students are important in improving their mathematical modelling competency.

Mediating Effects of the Four Mastery Goal Components on Relationships between Metacognition and Mathematical Modelling Competency

We expected that task-approach, task-avoidance, self-approach, and self-avoidance goals have mediating effects on the relationship between metacognition and mathematical modelling competency. Table 3 shows the mediating effect analysis results of the four mastery goal components.

<table>
<thead>
<tr>
<th>Mediator</th>
<th>$z$</th>
<th>$P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MC $\rightarrow$ TAP $\rightarrow$ MMC</td>
<td>2.63</td>
<td>0.008</td>
</tr>
<tr>
<td>MC $\rightarrow$ TAV $\rightarrow$ MMC</td>
<td>-3.33</td>
<td>0.000</td>
</tr>
<tr>
<td>MC $\rightarrow$ SAP $\rightarrow$ MMC</td>
<td>1.85</td>
<td>0.064</td>
</tr>
</tbody>
</table>


Mediation effects were determined using the Sobel test to confirm the mediating effect of the four mastery goal components. Task-approach goals ($z = 2.63, p < 0.05$) and task-avoidance goals ($z = -3.33, p < 0.05$) are significant partial mediators for mastery goals on mathematical modelling competency. Self-approach goal ($z = 1.85, p > 0.05$) is a significant full mediator for mastery goals on mathematical modelling competency. Self-avoidance goal is not a mediator for mastery goals on mathematical modelling competency. Therefore, H3 is confirmed and metacognition has a direct significant effect on mathematical modelling competency ($\beta = 0.441, t = 5.106, p < 0.05$).

Discussion

Application and modelling in a mathematics classroom have received strong support from several educational researchers in the last few decades (Niss et al., 2007); this development facilitated the examination of whether metacognition and mastery goal improve mathematical modelling competency. The purpose of the present study is to test the relationship between metacognition and mastery goal that
might influence mathematical modelling competency in students of mathematics education programmes. Considering the important role of mastery goal, research is surprisingly limited on how the four mastery goal components (i.e., task approach and task avoidance, self-approach, and self-avoidance) as a mediator on the relationship between metacognition and mathematical modelling competency.

According to the results of SEM, metacognition positively influences mathematical modelling competency. The expected positive effects of metacognition on students’ mathematical modelling competency corroborate previous research findings in mathematics achievement (Bonnett et al., 2016; Callan et al., 2016; Özcan, 2016; Tzohar-Rozen & Kramarski, 2014) and problem solving skills (Yusnaeni & Corebima, 2017). Other studies indicated that metacognition influences the modelling strategy development of students when the effects of four metacognitive components are taken into consideration (i.e., awareness, planning, cognitive strategy, and self-checking) (Yildirim, 2010). For instance, self-checking abilities, cognitive strategy, and planning skills mediate modelling competency development. Moreover, the adoption of metacognition in mathematical modelling classroom could create the use of common approaches, such as task analysis, task representation, prediction, planning, observing, checking, reflection, and evaluation of success (Pennequin, Sorel, Nanty, & Fontaine, 2010). Lingefjärd (2011) confirmed that metacognitive competencies are highly important to be involved in a model for barriers and chances given that mathematical modelling competency is known as complex and difficult task.

SEM analysis shows significant and positive influence of task- and self-approach goal on mathematical modelling competency, whereas task-avoidance goal is significantly and negatively related to mathematical modelling competency. By contrast, self-avoidance goals did not affect mathematical modelling competency. Our findings partially endorsed prior studies in which (1) task-based goals predicted material absorption in class (Elliot et al., 2011), deep learning (Soltaninejad, 2015), and effective strategy use (Wynne, 2014) and (2) self-approach goals are predictors of academic self-concept (Méndez-Giménez et al., 2017). One possible reason for this finding is perception of ability. Students who hold a task-approach goal are more likely to accomplish tasks well or in other words they could adopt the absolute demands of the task as the evaluative referent. At the same time, students who utilise a self-approach goal intend to improve their performance by comparing what they have done before or they focus on intrapersonal trajectory as the evaluative referent. Therefore, task- and self-based goals were correlated to mastery-based goals. The presence of a mastery goal in mathematical modelling classroom would endorse students–teacher relationship as well as peer inclusion and conflict (Polychroni et al., 2012) because modelling activity refers to group work as a means to find solutions.
Compared with task- and self-approach goal, the current discoveries found that task-avoidance goals are significantly and negatively related to mathematical modelling competency and self-avoidance goals did not affect their mathematical modelling competency. Our findings would appear to corroborate previous research (Elliot & McGregor, 2001; Howell & Watson, 2007; Karabenick, 2003; Moller & Elliot, 2006; Witkow & Fuligni, 2007; Yang et al., 2016), which stated that student who focus on mastery-avoidance goals have more negative association with academic performance, have no relations with deep processing, are more anxious, and need more help-seeking. One possible reason for this negative relationship is that students who focus on avoidance of task and self-intend to avoid misconceptions, not learning, or not mastering task. This result can be explained by Elliot et al. (2011) who confirmed that children adopting avoidance-based goals usually concentrate on failure, and regulations have to keep away from this negative probability.

The current study further found that task-approach and task-avoidance goals are partial mediators that improve the causal relationship between metacognition and mathematical modelling competency. However, task-approach goals are positive partial mediators and task-avoidance goals are negative partial mediators. In addition, self-approach goals are positive full mediators, whereas self-avoidance goals are not mediators between metacognition and mathematical modelling competency. Task-approach, task-avoidance, and self-approach goals may be meaningful factors that associate students’ metacognition and their mathematical modelling competency. These findings are consistent with previous research (Chen, 2015; Diseth & Kobbeltvedt, 2010) that found mastery goals as mediators in academic achievement. This result can be clarified by earlier research, which stated a positive correlation between mastery goal and metacognition (Gardner, Jabbour, Williams, & Huerta, 2016; Gul & Shehzad, 2012; King & Mcinerney, 2016; Mirzaei, Phang, Sulaiman, Kashefi, & Ismail, 2012). Hence, students who focus on mastery goals are usually self-regulated, self-monitoring, and apply organizational strategies on certain tasks. This finding implies that the presence of task- and self-approach goals in the mathematical modelling classroom would strengthen their metacognitive activity, whereas task-avoidance goals would deteriorate their metacognition, which influences mathematical modelling competency.

**Conclusions and Suggestions**

The findings of present research provide further evidence that mastery goals are observed to have positive effects on mathematical modelling competency. SEM analysis found significant and positive influences of task-and self-approach goals on mathematical modelling competency, whereas task-avoidance goals are significantly and negatively related to mathematical modelling competency. By contrast, self-
avoidance goals did not influence mathematical modelling competency. The task-
approach goal is positive partial mediator, task-avoidance goal is negative partial
mediator, self-approach goal is positive full mediator, and self-avoidance goal is
not a mediator between metacognition and mathematical modelling competency.
By summarizing these results, we argue that metacognition is a powerful factor that
can be influenced by mastery goal, and in turn, influence mathematical modelling
competency. Our results recommend examination of the effects of metacognition
and mastery goal toward every single sub-construct of mathematical modelling
competency, which may make stronger analysis. Future research should examine the
effect of metacognition and mastery goal using an experimental study because the
current study cannot explain causal effect between these variables.

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